January 10, 2001 WOCOMAL Varsity Meet

ROUND I: Similarity and Pythagorean theorem

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS DIRECTED IN THE PROBLEM

1. If five foot- one inch tall Elaine casts an 84 inch shadow, then how tall is her friend Jerry if he has a shadow at the same time which is one foot shorter than hers? Give your answer in feet and nearest inch.

2. The altitude to the hypotenuse of a right triangle has length 10 and divides the hypotenuse into two pieces, one 21 units longer than the other. Find the length of the shorter of these two pieces.

3. Let $\overline{CB}$ be a leg of the right triangle of least perimeter whose sides have integral lengths, whose hypotenuse is one unit longer than CB, and in which CB ≥ 100. Find CB.

ANSWERS

(1 pt) 1. ______________________________

(2 pts) 2. ________________

(3 pts) 3. ________________

Mass. Academy, Tahanto, Westborough

January 10, 2001 WOCOMAL Varsity Meet

ROUND II: Algebra 1 - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS DIRECTED IN THE PROBLEM

1. Solve: $12\left(\frac{1}{x-2}\right)=32\left(\frac{1}{x+8}\right)$

2. A person born in the year x^2+73 celebrated her 14th birthday in the year $(x+1)^2$. In what year was she born?

3. Ann is twice as old as Judy was when Ann was as old as Judy is now. If the sum of their present ages is 56, how old is each now?

ANSWERS

(1 pt) 1. $x =$ ____________________

(2 pts) 2. ____________________

(3 pts) 3. ANN ________ JUDY ________

Quaboag, South, Tantasqua

January 10, 2001

WOCOMAL Varsity Meet

ROUND III: Functions

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS DIRECTED IN THE PROBLEM

1. For $y = \dfrac{5x-3}{4}$, find the formula for the inverse function in the form y = an expression involving x.

2. If f(4x+3) = 2x+1, find f(-9).

3. Find all values of x for which f(f(x)) = 11f(x), given that f(x) = $x^2 - x$.

ANSWERS

(1 pt) 1. y = ______________________

(2 pts) 2. ___________

(3 pts) 3. ______________________

Doherty, Hudson, Shrewsbury

ROUND IV: Combinatorics

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS DIRECTED IN THE PROBLEM

1. There are 10 points in a plane, no three of which are collinear. How many triangles can be formed having the given points as vertices?

2. How many odd integers between 1000 and 10,000 are written without the digit 6?.

3. George set type by hand for a book of poems which he wrote. After setting the type for the poems, one poem to a page, he realized that the pages should be numbered starting with 1. Starting at midnight, he set all the page numbers in their proper places. When he finally finished the job, it was 1:45 am. If it took him $\frac{1}{2}$ minute to set a single digit properly on a page, how many poems are in the book?

ANSWERS

(1 pt) 1. ____________________

(2 pts) 2. ____________________

(3 pts) 3. ____________________

Bancroft, Shrewsbury, Tahanto

January 10, 2001 WOCOMAL Varsity Meet

ROUND V: Analytic geometry of straight lines and conic sections

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS DIRECTED IN THE PROBLEM

1. What equality will make the lines with these equations parallel?

   ax + by = c and x-d = e(y-f)

2. The area of an ellipse is given by the expression πab where a and b are half the lengths of the major and minor axes. Fine the area of the ellipse with equation

   $4x^2 + 9y^2 - 54y + 45 = 0$.

3. Find the coordinates of the focal point of the parabola with equation $16y = x^2 - 4x + 52$.

ANSWERS

(1 pt) 1. ______________________

(2 pts) 2. ____________

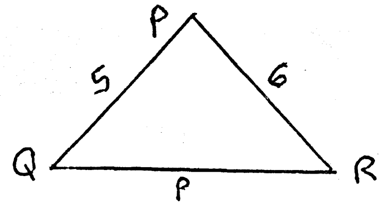
(3 pts) 3. (,)

Auburn, Bancroft, QSC

TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS DIRECTED IN THE PROBLEM and ON THE SEPARATE TEAM ROUND ANSWER SHEET

1. Write a condition in terms of side length p so that ΔPQR is acute.

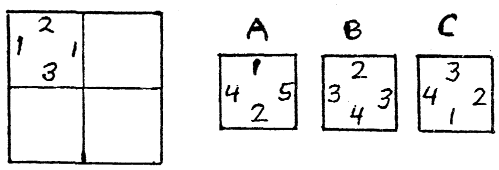


2. Mr. T's December salary was \$D. He got a 10% raise for the month of January, but he received 10% less in February than in January. In terms of D, what was Mr.T's average salary for the 3 months?

3. Evaluate f(10,6) given:

$$f(x,y)=\begin{cases} f(x-y,y-1)+2 & \text{when } x>y>0 \\ f(y-x,x-1)+1 & \text{when } y>x>0 \\ 10 & \text{otherwise} \end{cases}$$

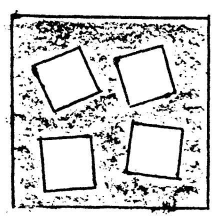
4. How many diferent ways can tiles A, B, and C be placed in the open squares such that bordering numbers are equal? All three tiles must be used for each solution. Tiles may be rotated but not flipped over.



5. The circle with equation $(x-4)^2+(y-2)^2=52$ passes through A(0,8). How many distinct chords of the circle have endpoint A and integer lengths?

6. Solve for x: $|x-2|^2+3|x-2|=4$

7. A side of the large square is equal to the square of a side of each small square. The area of the shaded region is 60. Find the side of the large square.



8. For n a positive integer, let $f(n)=$ n(n+1)(n+2). Find n if $\dfrac{f(8)}{f(n)}=f(4)$.

9. Of 60 people in a room, $\frac{2}{3}$ are women and $\frac{2}{5}$ of the people have blonde hair. What are the maximum and minimum numbers of women in the room whose hair is not blonde ?

January 10, 2001 WOCOMAL Varsity Meet ANSWERS

ROUND I sim Pyth

1. 1 pt 4 ft 4 in $4'4''$

2. 2 pts 4

3. 3 pts 112

ROUND II alg 1

1. 1 pt 8

2. 2 pts 1922

3. 3 pts ANN 32 JUDY 24

ROUND III funct

1. 1 pt $y = \frac{4x+3}{5}$ OR $\frac{4}{5}x + \frac{3}{5}$

2. 2 pts -5

3. 3 pts $-3, 0, 1, 4$ any order

ROUND IV comb

1. 1 pt 120

2. 2 pts 3240

3. 3 pts 106

ROUND V analyt

1. 1 pt $\frac{-a}{b} = \frac{1}{e}$ OR equivalent equation

2. 2 pts 6π

3. 3 pts $(2, 7)$

TEAM ROUND 2 pts each

1. $\sqrt{11} < p < \sqrt{61}$ OR $11 < p^2 < 61$

2. $1.03D$ OR $\frac{103D}{100}$ OR 103% of D

3. 14

4. 0 OR none

5. 28

6. 1 & 3 need both

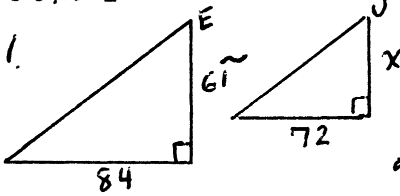
7. 10

8. 1

9. MAX 36 MIN 16

January 10, 2001 WOCOMAL VARSITY MEET BRIEF SOLUTIONS

ROUND I

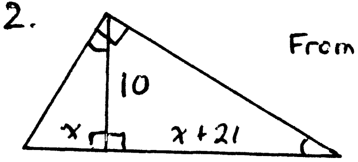
1. 

$$\frac{x}{61} = \frac{72}{84}$$

$x = 52.2857$ in

or $48 + 4$ in

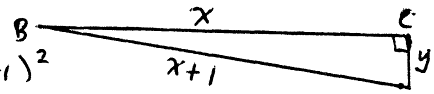
$\therefore$ 4 ft 4 in

2. 

From ~ Δ's or theorem

$$\frac{x}{10} = \frac{10}{x+21}$$

$$x^2 + 21x - 100 = (x+25)(x-4) = 0, \quad x = 4$$

3. 

$x^2 + y^2 = (x+1)^2$

$y^2 = 2x + 1$ with $x \geq 100$

and $2x+1$ a perfect square

$\therefore 2x + 1 = 225$ and $x = 112$

ROUND II

1. $12(x+8) = 32(x-2)$. May $\div 4$

$3x + 24 = 8x - 16$

$40 = 5x$ and $x = 8$

2. $x^2 + 73 + 14 = (x+1)^2$

$87 = 2x + 1$

$43 = x$

Then $43^2 + 73 = 1922$

3.

	now	then
Ann	a	j
Judy	j	$j-(a-j)$

} same difference between ages now and then

$a + j = 56$ and $a = 2(2j - a)$

$a + \frac{3}{4}a = 56 \leftarrow 3a = 4j$

$a = 32$ and $j = 24$

ROUND III

1 Linear so switch x and y and solve for y

$x = \dfrac{5y-3}{4}$

$4x = 5y - 3$

$4x + 3 = 5y$ and $y = \dfrac{4x+3}{5}$

ROUND III cont.

2. Set $4x + 3 = -9$ and get $x = -3$.

Then $f(-9) = 2(-3) + 1 = -5$

3. $(x^2-x)^2 - (x^2-x) = 11(x^2-x)$

$(x^2-x)[x^2 - x - 1 - 11] = 0$

$x(x-1)(x^2 - x - 12) = 0$

$x(x-1)(x-4)(x+3) = 0$

$x = 0, 1, 4, -3$

ROUND IV

1 $_{10}C_3 = \dfrac{10!}{3!\,7!} = 120$

2. 4 digits, first not 0, none is 6, 4th is odd

$8 \cdot 9 \cdot 9 \cdot 5 = 3240$

3 midnight to 1:45 am is 105 minutes or 210 half minutes

The first 9 poem take 9 half min

The next 90 2 digits, take 180 half min.

$210 - 189 = 21$ half min remain for 3-digit numbers and $21 \div 3 = 7$.

$9 + 90 + 7 = 106$ poems

ROUND V

1 Make slopes = Use $y = mx + b$ form

$ax + by = c \Rightarrow y = -\frac{a}{b}x + \frac{c}{b}$

$x - d = e(y - f) \Rightarrow y = \frac{1}{e}x$ + other terms

$\therefore -\dfrac{a}{b} = \dfrac{1}{e}$

2. $4x^2 + 9y^2 - 54y = -45$

$4x^2 + 9(y^2 - 6y + 9) = -45 + 81$

$4x^2 + 9(y-3)^2 = 36$

$\dfrac{x^2}{9} + \dfrac{(y-3)^2}{4} = 1 \Rightarrow a = 3, b = 2$

Area $= \pi ab = 6\pi$

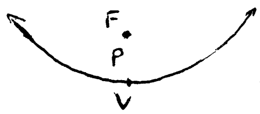
ROUND V cont

3. $16y = x^2 - 4x + 4 + 48$

$4py = (x-2)^2 + 48$ and $p = 4$

$x = 2$ makes $16y = 48$ and $y = 3$

Vertex $(2,3)$ and opens upward



Focal pt $(2,7)$

p above the vertex

TEAM ROUND

1. For acute $\angle P$ need $p^2 < 5^2 + 6^2$ or $p^2 < 61$

" " $\angle R$ " $5^2 < p^2 + 6^2$ or $-11 < p^2$

" " $\angle Q$ " $6^2 < p^2 + 5^2$ or $11 < p^2$

Thus $11 < p^2 < 61$ or $\sqrt{11} < p < \sqrt{61}$

2. Dec salary $= D$

Jan " $= 1.1D$

Feb " $= .9(1.1D) = .99D$

Ave: $\dfrac{D + 1.1D + .99D}{3} = 1.03D$

3 $f(10,6) = f(4,5) + 2$ since $10 > 6 > 0$

$= f(1,3) + 1 + 2$ " $5 > 4 > 0$

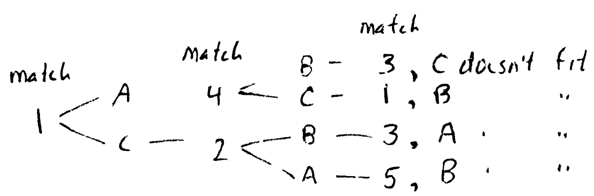
$= f(2,0) + 1 + 3$ " $3 > 1 > 0$

$= 10 + 4$ " otherwise

$= 14$

4 Trial without success suggest none

This organized search will verify that.



5 radius $= \sqrt{52}$, diam $= 2\sqrt{52} \approx 14.422$

Chords with endpt A can have lengths 1, 2, 3, …, 14 and there are 2 of each length. Thus 28

TEAM ROUND cont

6 $|x-2|^2 + 3|x-2| - 4 = 0$

$(|x-2| + 4)(|x-2| - 1) = 0$

$\underbrace{|x-2| = -4}_{\text{NO}}$ or $\underbrace{|x-2| = 1}_{\text{OK}}$

$x = 3$ or 1

7 Let x = side of small square

Then x^2 = " " large "

Area: large sq − 4 small sq = 60

$x^4 - 4x^2 = 60$

$x^4 - 4x^2 - 60 = 0$

$(x^2 - 10)\underbrace{(x^2 + 6)}_{\neq 0} = 0$

$x^2 = 10$, ans

8 $\dfrac{f(8)}{f(n)} = f(4) \Rightarrow f(n) = \dfrac{f(8)}{f(4)}$

which becomes

$n(n+1)(n+2) = \dfrac{8 \cdot 9 \cdot 10}{4 \cdot 5 \cdot 6} = 6$

or $n(n+1)(n+2) = 1 \cdot 2 \cdot 3$ and $n = 1$

9 $\frac{2}{3}$ of $60 = 40$ women; $\frac{2}{5}(60) = 24$ blondes

MAX

	W	not W	
blonde	4	20	24
not blon	$\boxed{36}$	0	36
	40	20	60

Fill in "outside" numbers first

MIN

	W	not W	
blonde	24	0	24
not b	$\boxed{16}$	20	36
	40	20	60